

Isospin invariant description of double-beta decay in the IBM

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*supported by FUSTIPEN

**Marie Curie fellow

Franco's fest, Yale, October 2018

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Motivation

Nucleon-pair shell model

Effective operators in a collective subspace

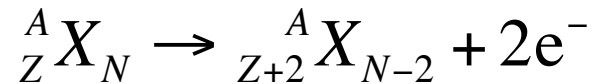
Mapping to boson operators

Application to $0\nu\beta\beta$ decay in the pf shell

Conclusions and outlook

Neutrino-less $\beta\beta$ decay

The process:



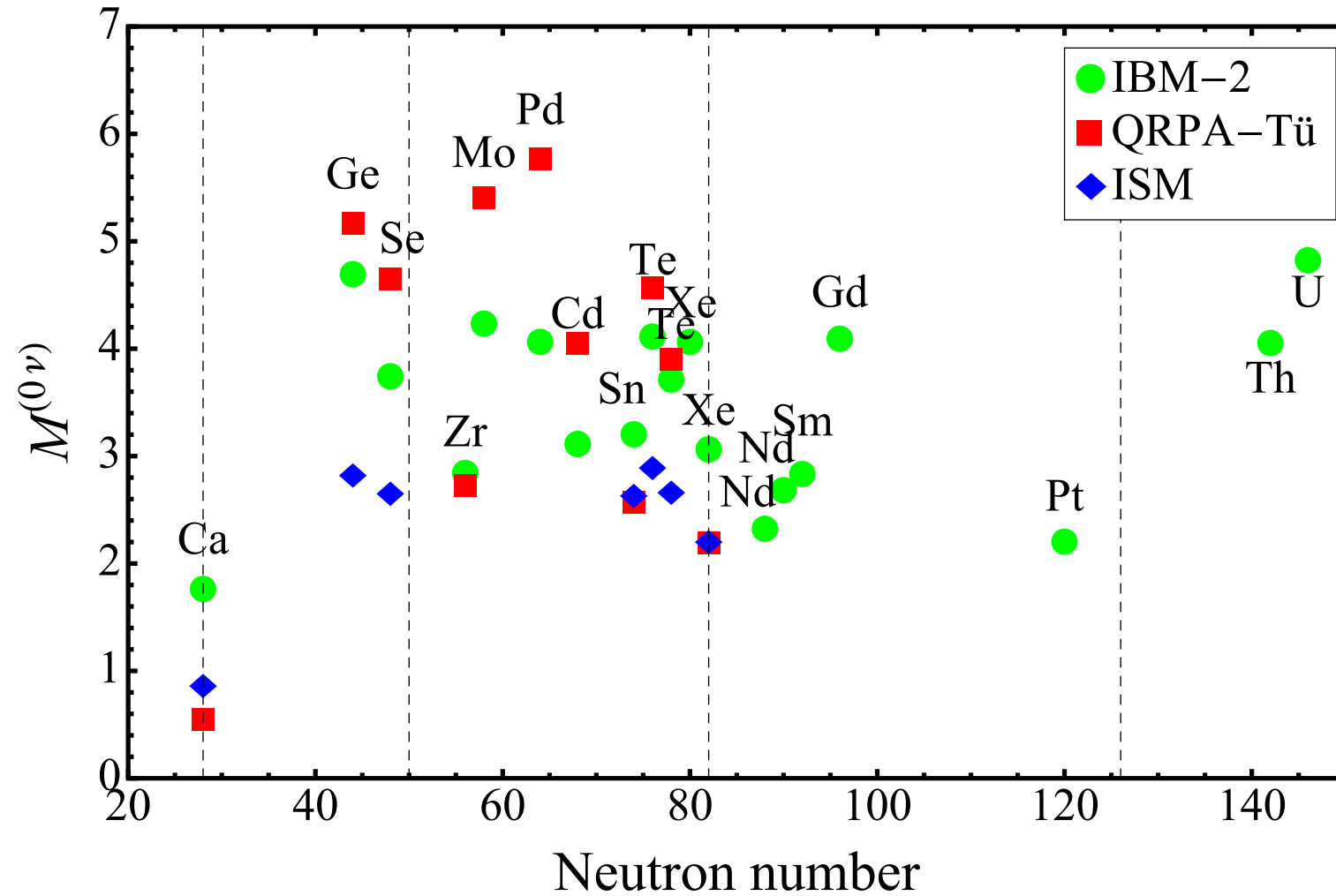
Importance: neutrinos are Majorana particles with mass, violation of lepton number, physics beyond the standard model,...

The half-life of this process is

$$\frac{1}{\tau_{1/2}^{0\nu}} = G_{0\nu} |M_{0\nu}|^2 |f(m_i, U_{ei})|^2$$

Nuclear physics must provide the nuclear matrix element $M_{0\nu}$.

Comparison SM-QRPA-IBM



Aims of this work

To obtain a better understanding of the relation between IBM and SM.

To use an isospin-invariant version of the IBM.

To study the influence of neutron-proton pairing on double-beta decay.

Nucleon-pair shell model (NPSM)

Pairs of fermions

$$P_{j_1 j_2 JM}^+ = \left(a_{j_1}^+ \times a_{j_2}^+ \right)_M^{(J)} \equiv P_{\alpha JM}^+$$

Basis states for $2n$ nucleons in NPSM

$$|\alpha_1 J_1 \dots \alpha_n J_n; L_2 \dots L_n\rangle \equiv \left(\dots \left(\left(P_{\alpha_1 J_1}^+ \times P_{\alpha_2 J_2}^+ \right)^{(L_2)} \times P_{\alpha_3 J_3}^+ \right)^{(L_3)} \times \dots \times P_{\alpha_n J_n}^+ \right)^{(L_n)} |O\rangle$$

Isospin-invariant formulation: $J \rightarrow JT$.

Matrix elements can be calculated with a recursive technique.

Collective subspace

Collective pairs of fermions

$$B_{JM}^+ = \sum_{j_1 j_2} \alpha_{j_1 j_2}^J \left(a_{j_1}^+ \times a_{j_2}^+ \right)_M^{(J)}$$

Collective basis states for $2n$ nucleons

$$|J_1 \dots J_n; L_2 \dots L_n\rangle \equiv \left(\dots \left(\left(B_{J_1}^+ \times B_{J_2}^+ \right)^{(L_2)} \times B_{J_3}^+ \right)^{(L_3)} \times \dots \times B_{J_n}^+ \right)^{(L_n)} |O\rangle$$

Dimension of the collective subspace much smaller than that of the full shell-model space, $\omega \ll \Omega$.

Effective operators

Let $\mathbf{H}_p$ be the collective subspace, $\mathbf{H}_Q$ the excluded space and $\mathbf{H}=\mathbf{H}_p+\mathbf{H}_Q$ the full SM space.

The method of Okubo-Lee-Suzuki (OLS) defines an operator η that maps states in $\mathbf{H}_p$ to states in $\mathbf{H}_Q$ such that

$$\hat{\eta}(\hat{P}|E_k\rangle) = \hat{Q}|E_k\rangle, \quad k = 1, \dots, \omega, \quad |E_k\rangle \in \mathbf{H}$$

For small nucleon number η can be calculated exactly

$$\eta_{ri} = \left(\left(\mathbf{I}_\Omega - \mathbf{b}^T \times \mathbf{b} \right) \times \tilde{\mathbf{E}}^T \times \mathbf{d}^{-1} \right)_{ri}, \quad r = 1, \dots, \Omega, \quad i = 1, \dots, \omega$$

Mapping to bosons

Map fermion pairs to bosons

$$B_{JM}^+ = \sum_{j_1 j_2} \alpha_{j_1 j_2}^J \left(a_{j_1}^+ \times a_{j_2}^+ \right)_M^{(J)} \Rightarrow b_{JM}^+$$

Basis states for n bosons

$$|b_i^n\rangle \equiv \left(\cdots \left(\left(b_{J_1}^+ \times b_{J_2}^+ \right)^{(L_2)} \times b_{J_3}^+ \right)^{(L_3)} \times \cdots \times b_{J_n}^+ \right)^{(L_n)} |0\rangle$$

Corresponding NPSM basis is not orthogonal.

Mapping is based on the diagonalization of the overlap matrix.

Mapping to boson operators

For the mapping to 1+2-body boson operators we need the correspondence for $n=1$ & $n=2$:

$$n = 1 : |B_{JM}\rangle \Rightarrow |b_{JM}\rangle$$

$$n = 2 : |\bar{B}_i^2\rangle \Rightarrow |\bar{b}_i^2\rangle = \sum_{j=1}^{\omega} c_{ij} |b_j^2\rangle$$

Boson operators are defined through

$$\langle \bar{b}_i^n | \hat{T}^b | \bar{b}_j^n \rangle = \langle \bar{B}_i^n | \hat{T}^f | \bar{B}_j^n \rangle$$

Application to $0\nu\beta\beta$ decay

Shell-model Hamiltonian in the pf shell:

Modified Kuo-Brown KB3G

Collective separable approximation to it

Shell-model $0\nu\beta\beta$ -decay operator defined via its matrix elements:

$$M_{0\nu\beta\beta} = M_{0\nu\beta\beta}^{\text{GT}} - \left(\frac{g_V}{g_A} \right)^2 M_{0\nu\beta\beta}^{\text{F}} + M_{0\nu\beta\beta}^{\text{T}}$$

Mapping with different bosons

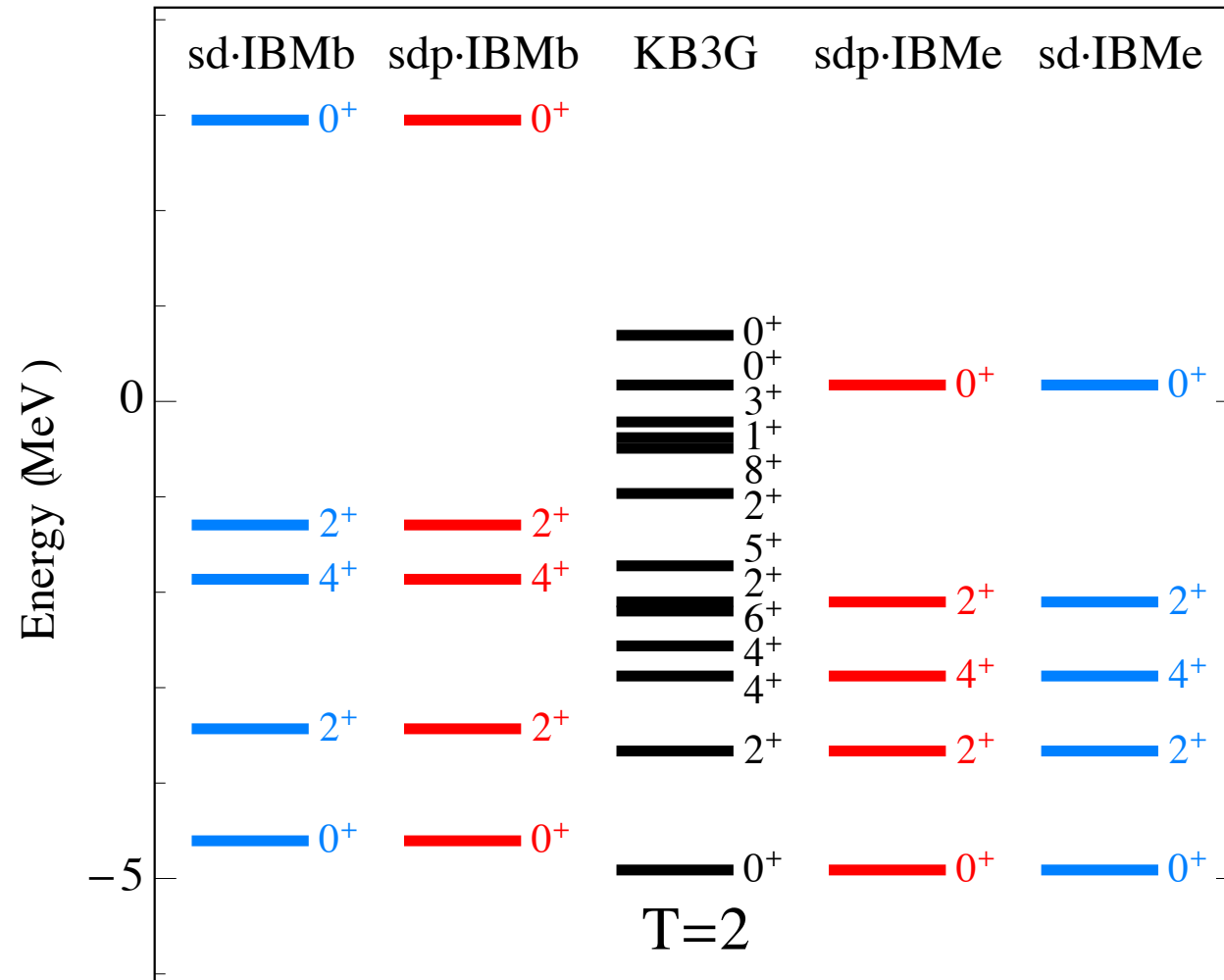
1. Bosons with $J=0$ (s) and $J=2$ (d) and isospin $T=1$.
This is isospin-invariant IBM (IBM-3).
2. In addition a boson with $J=1$ (p) and isospin $T=0$.
This to probe the importance of isoscalar correlations in $0\nu\beta\beta$ decay. This is referred to as p -IBM.

Mapping of

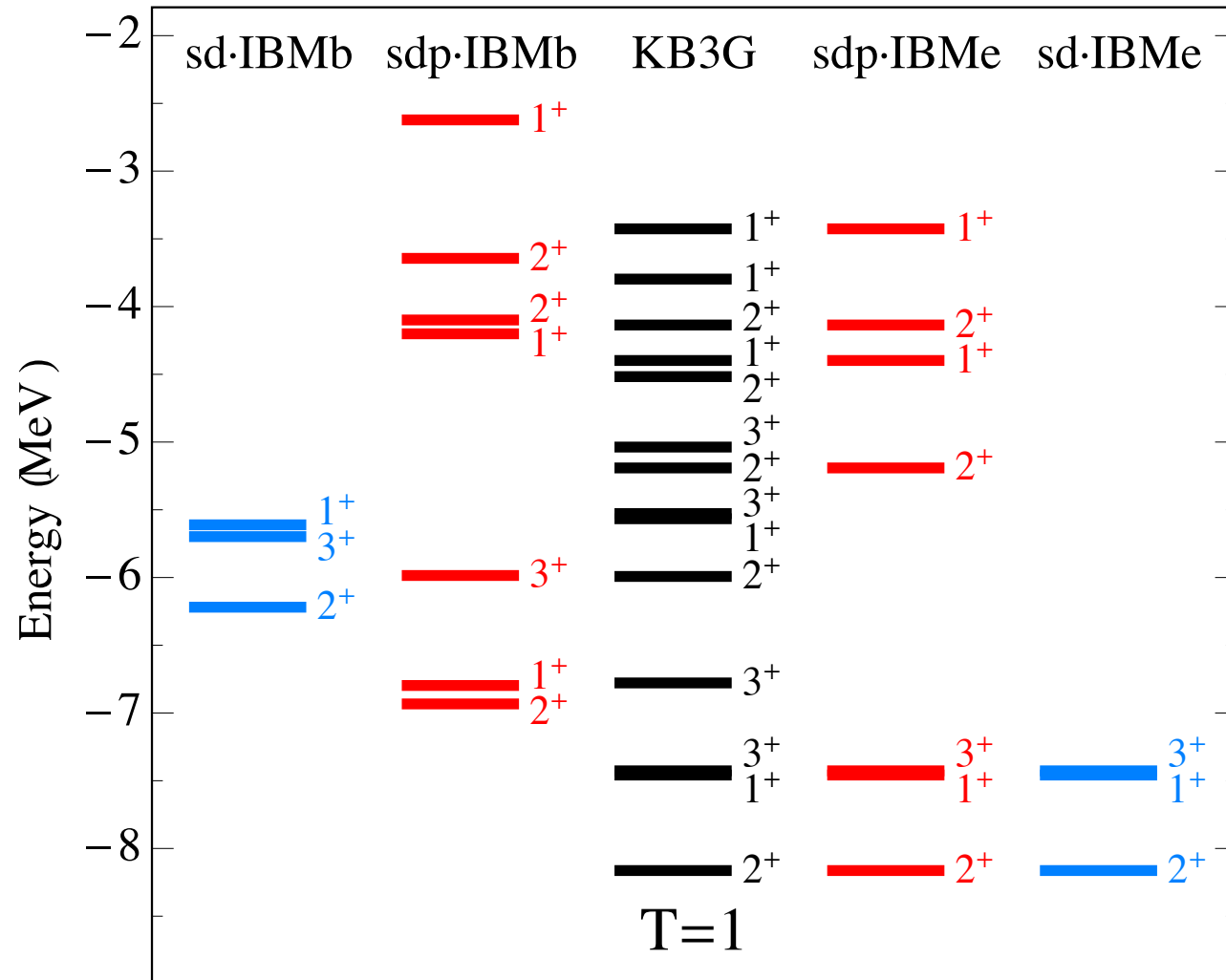
original SM operators $\rightarrow$ IBMb (bare)

effective SM operators $\rightarrow$ IBMe (effective)

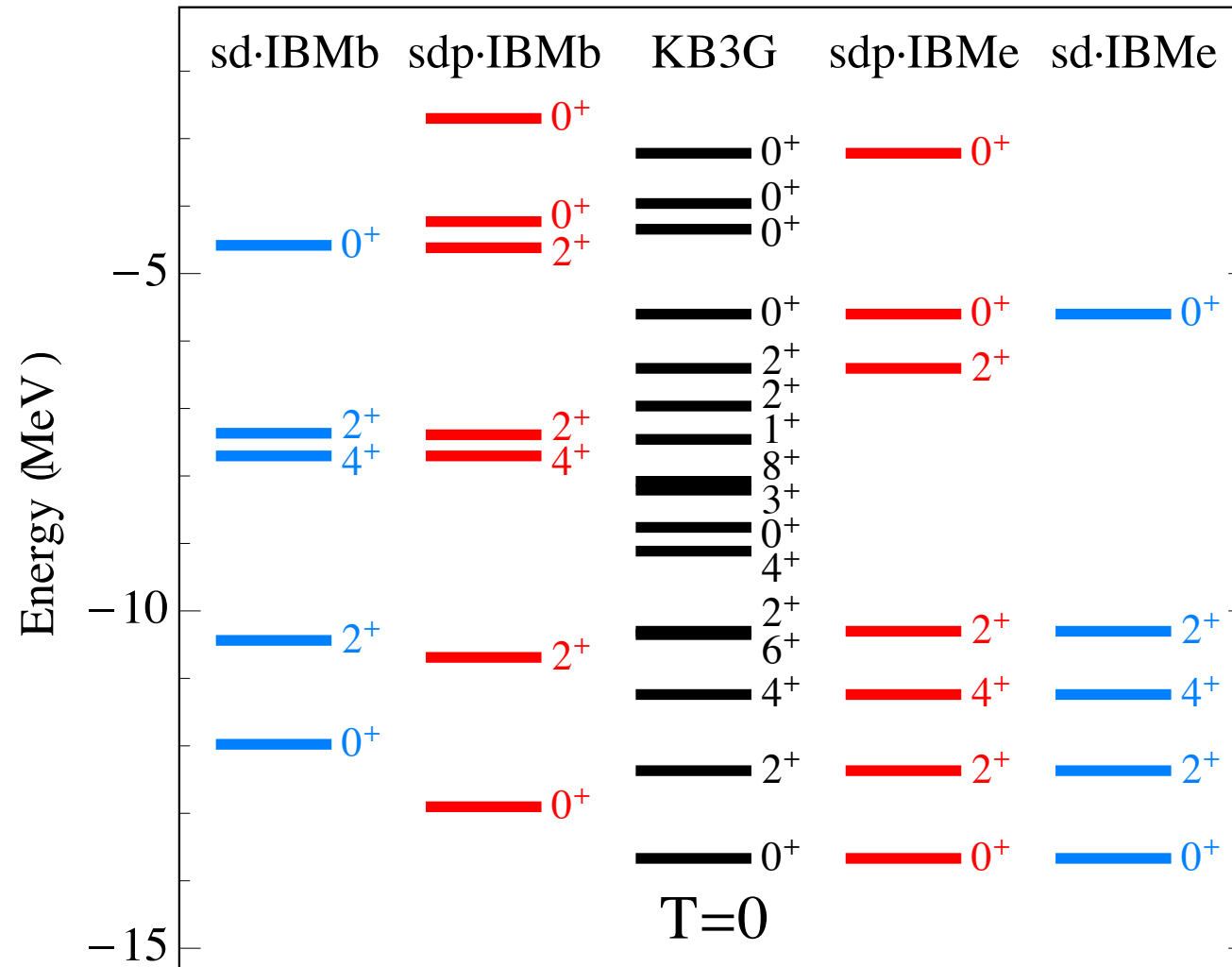
A=44 energy spectra



A=44 energy spectra



A=44 energy spectra



An A -dependent boson Hamiltonian

The mapped IBM Hamiltonian is obtained from
 $A=42$ and $A=44$.

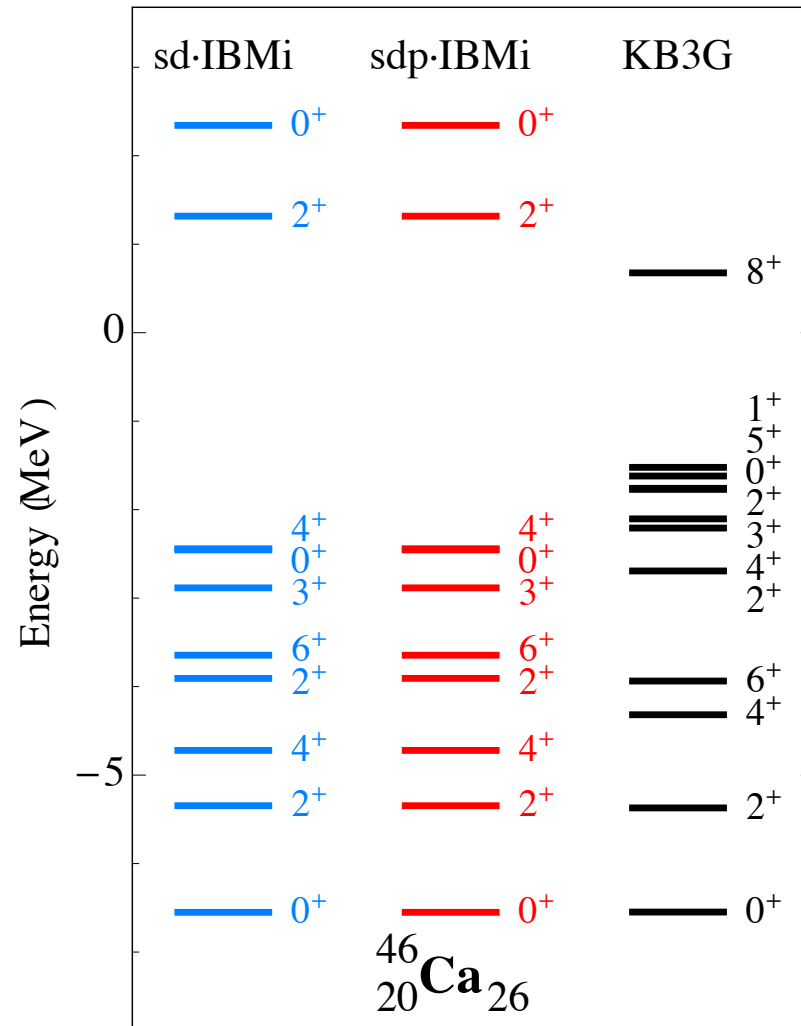
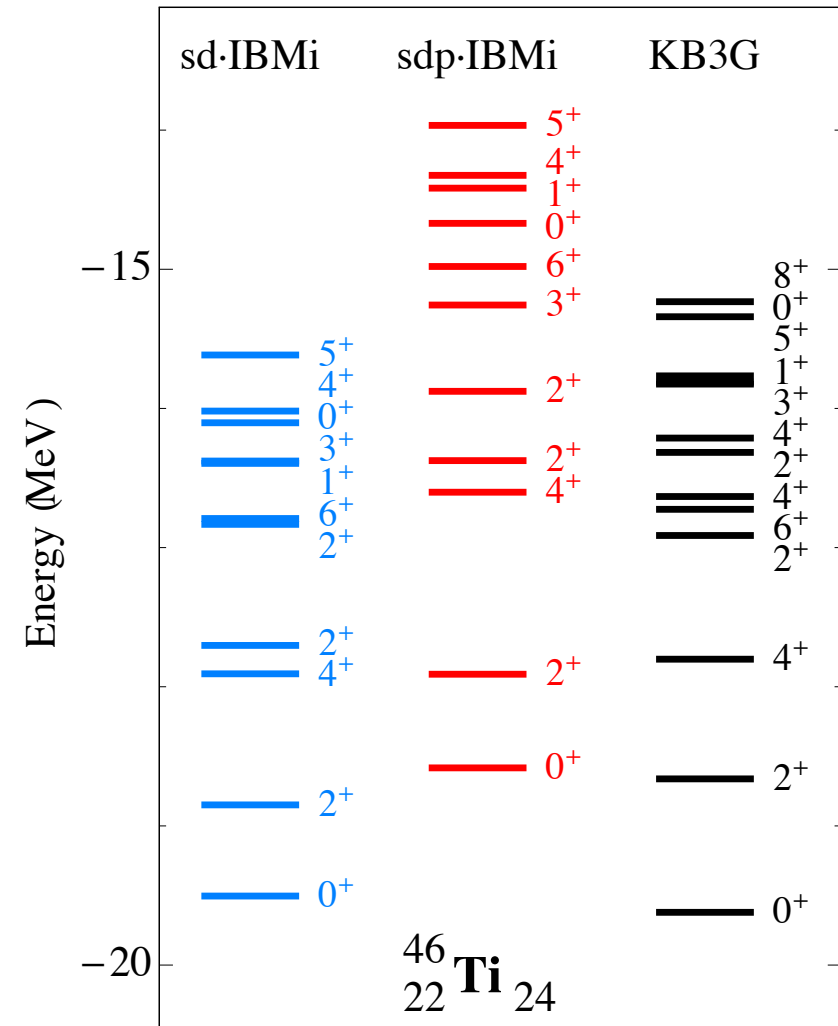
Use an (n, T) -dependent boson Hamiltonian obtained
by interpolation

$$\hat{H}^b = x\hat{H}_e^b + (1-x)\hat{H}_b^b$$

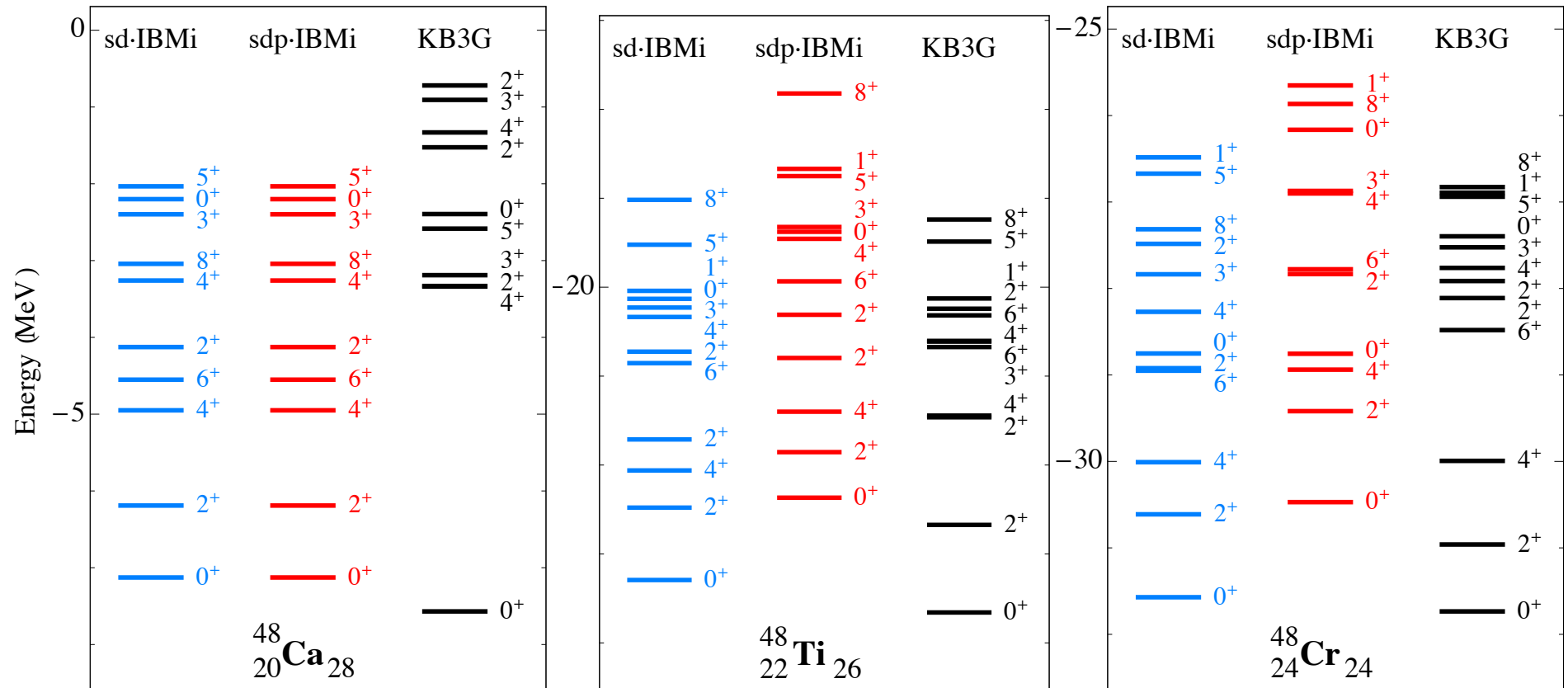
Based on the property

$$\left\langle \hat{H}_e^b \right\rangle_{n,J,T} \leq \left\langle \hat{H}^f \right\rangle_{2n,J,T} \leq \left\langle \hat{H}_b^b \right\rangle_{n,J,T}$$

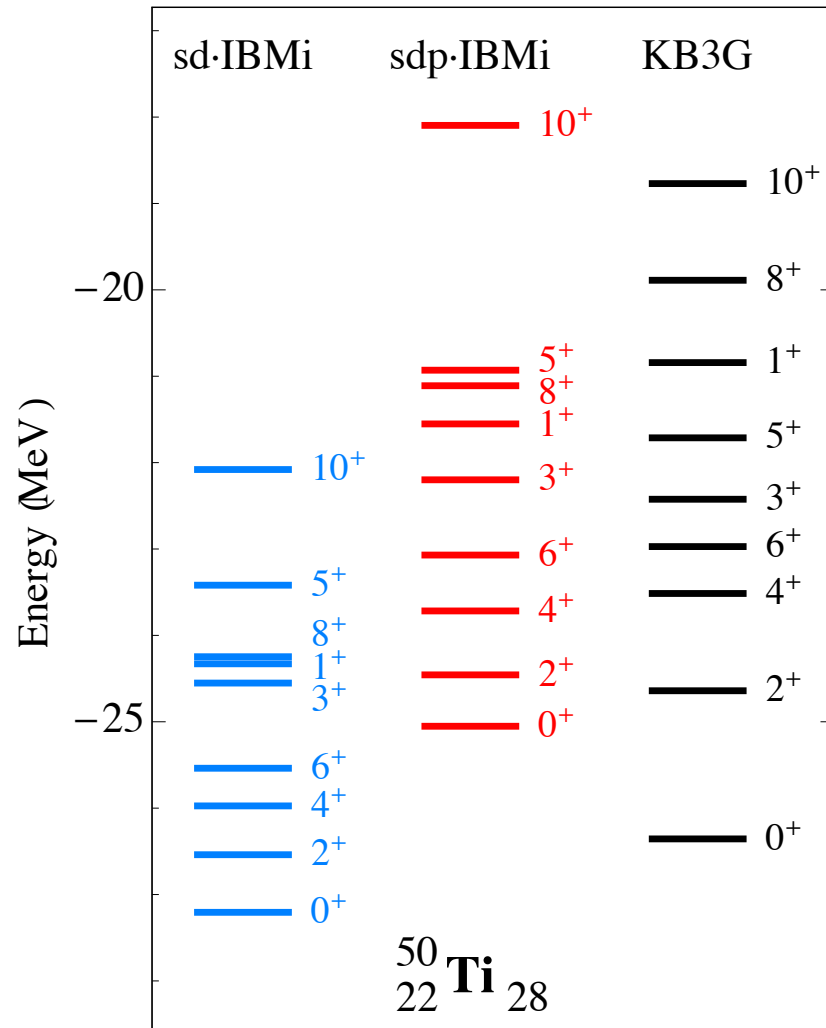
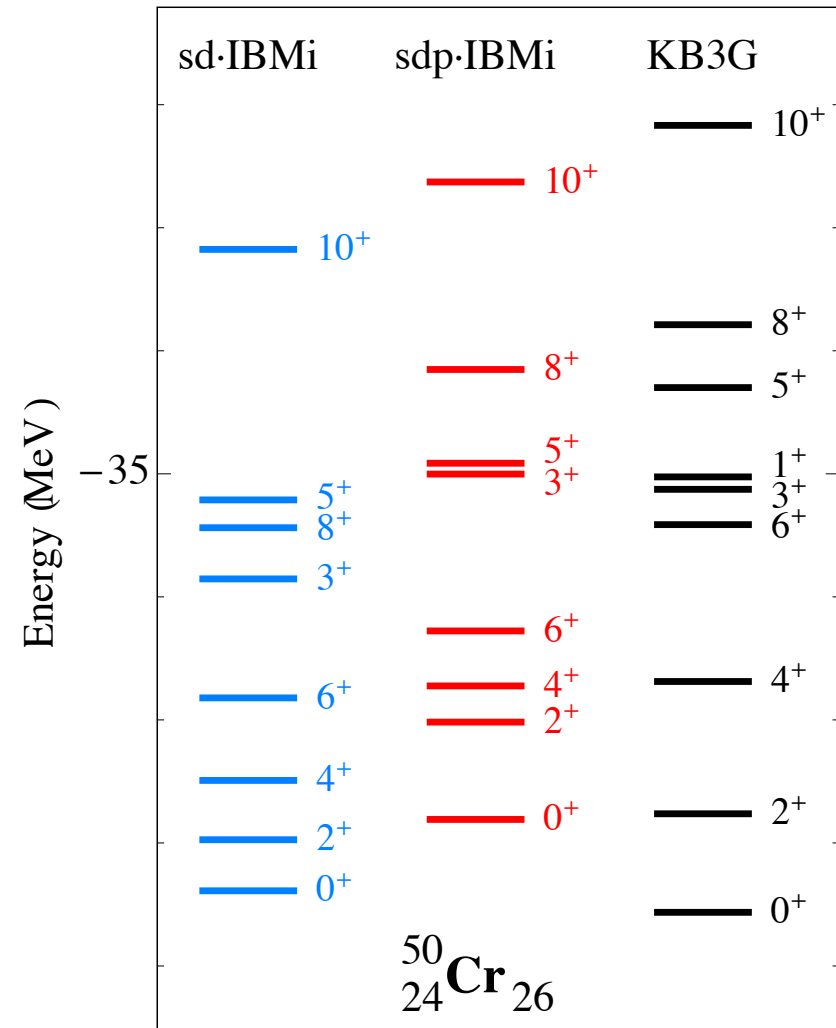
A=46 energy spectra



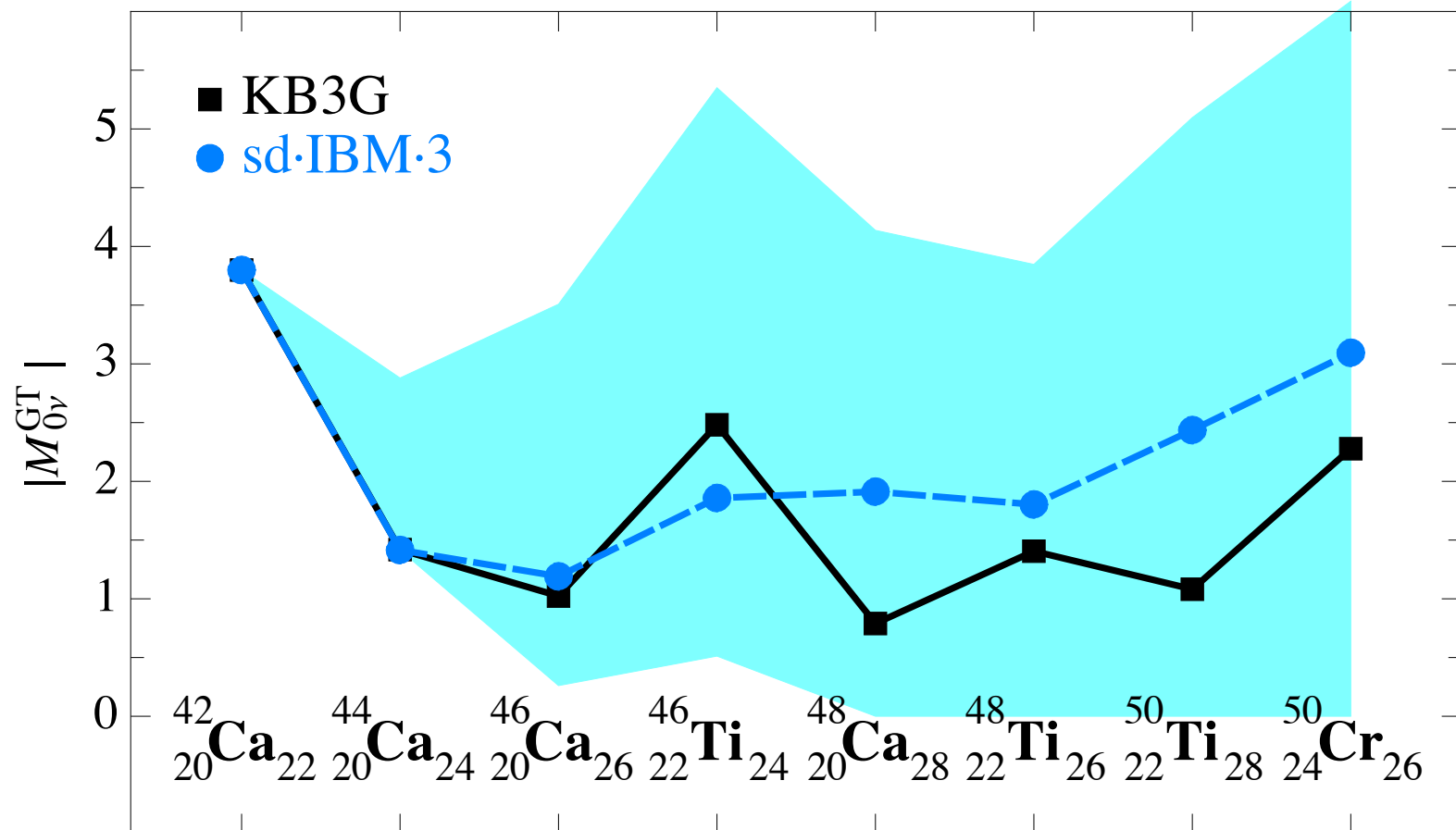
A=48 energy spectra



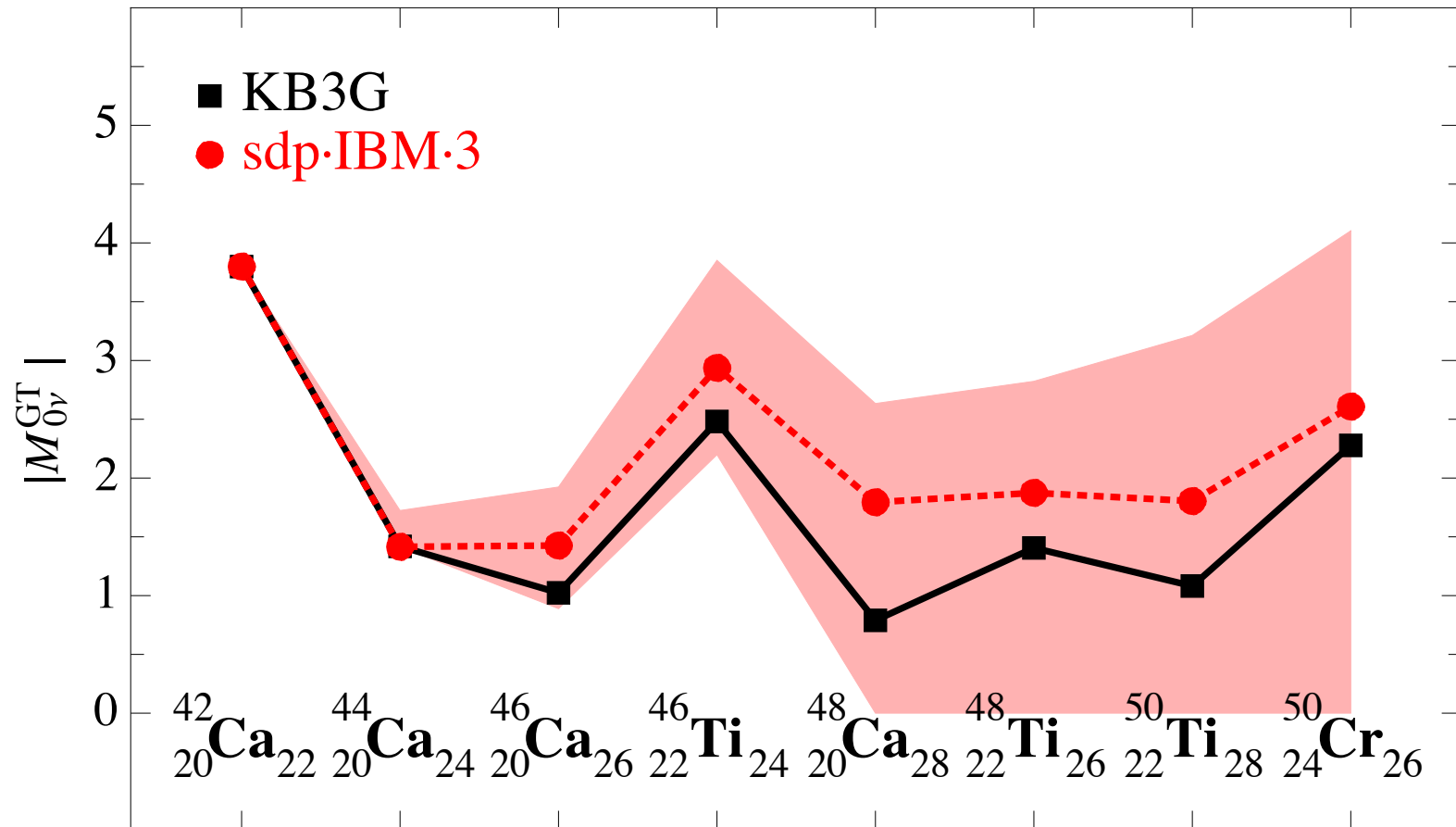
A=50 energy spectra



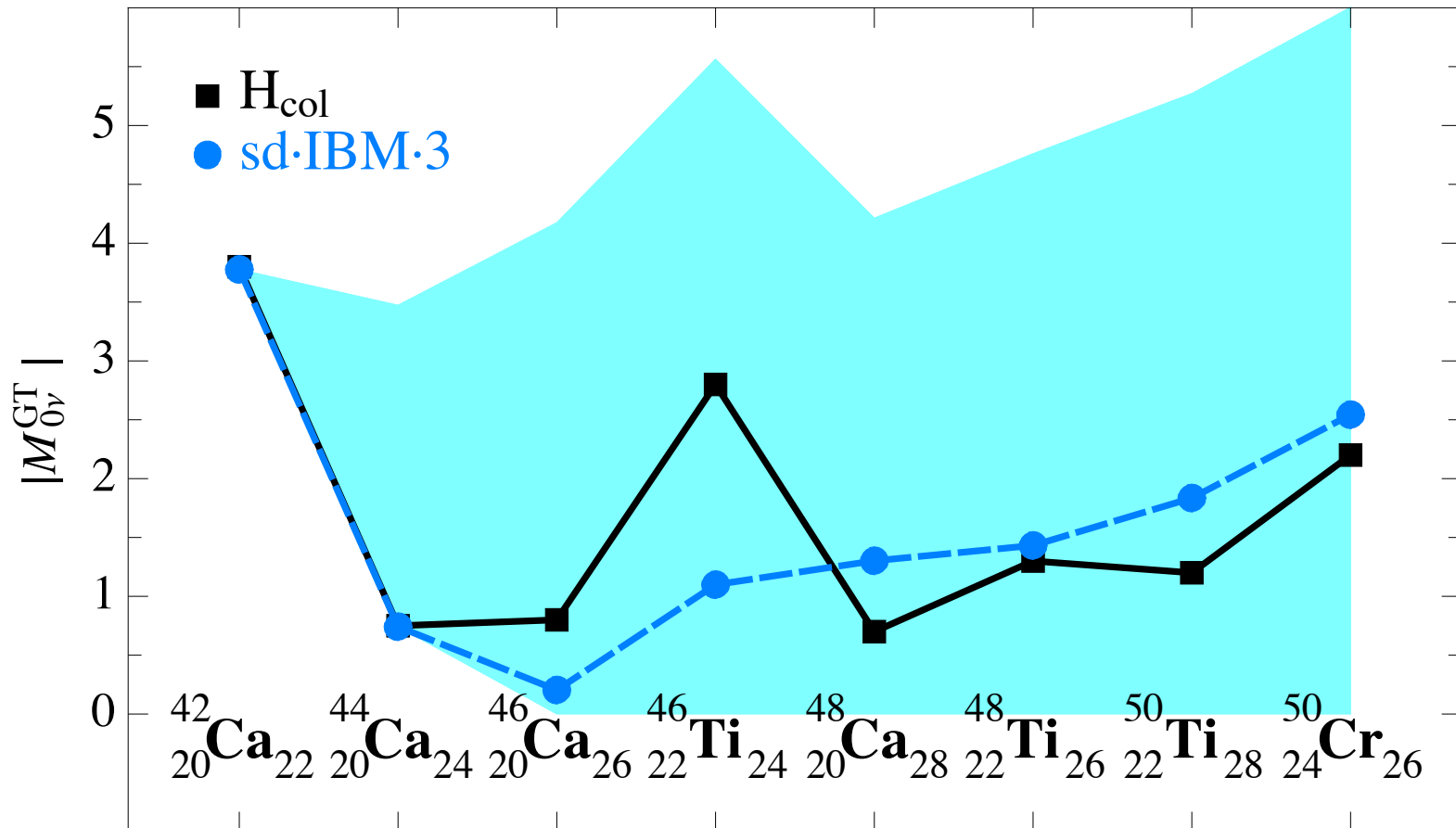
Gamow-Teller $0\nu\beta\beta$ in *sd*-IBM-3



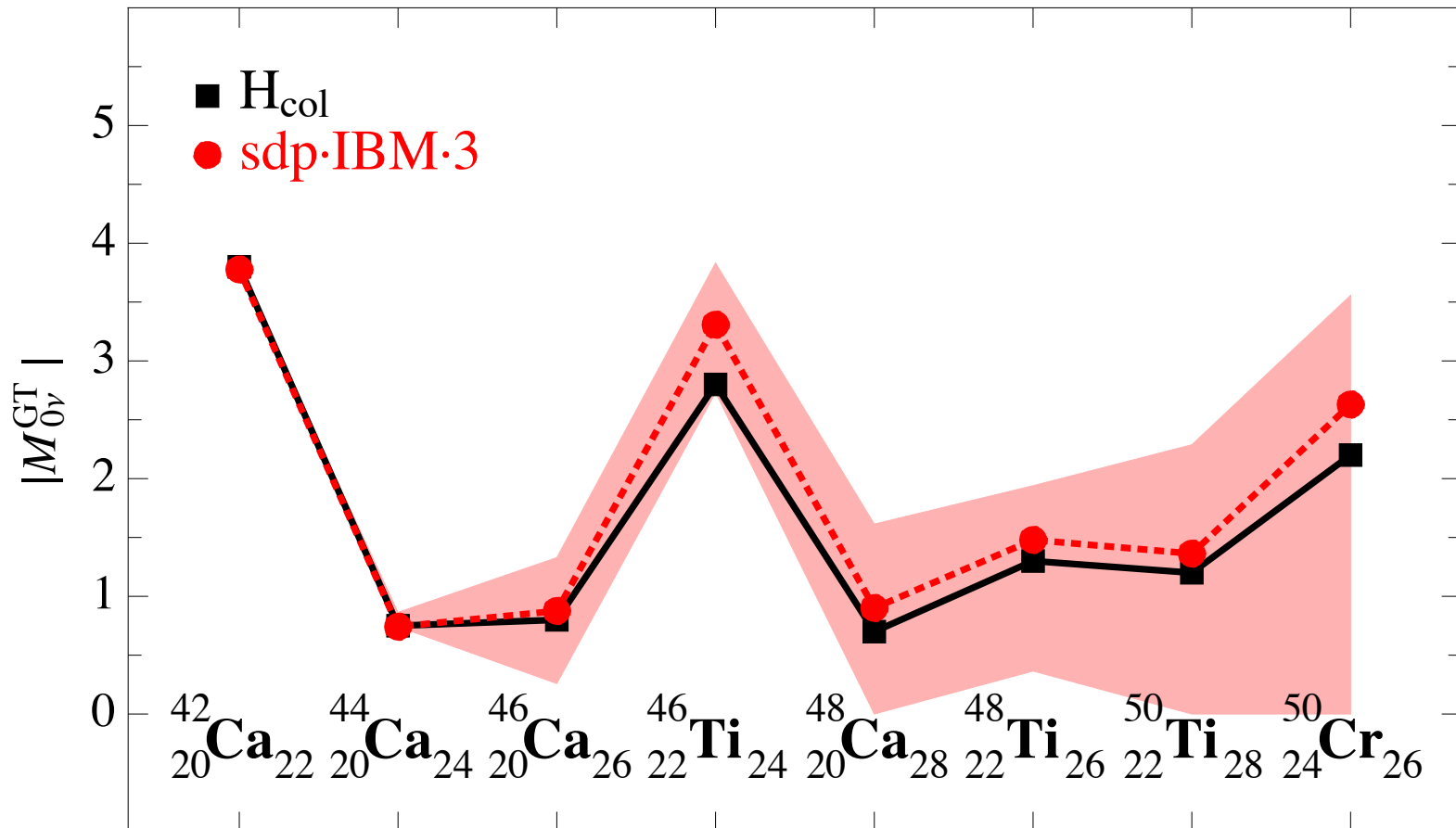
Gamow-Teller $0\nu\beta\beta$ in *sdp*-IBM-3



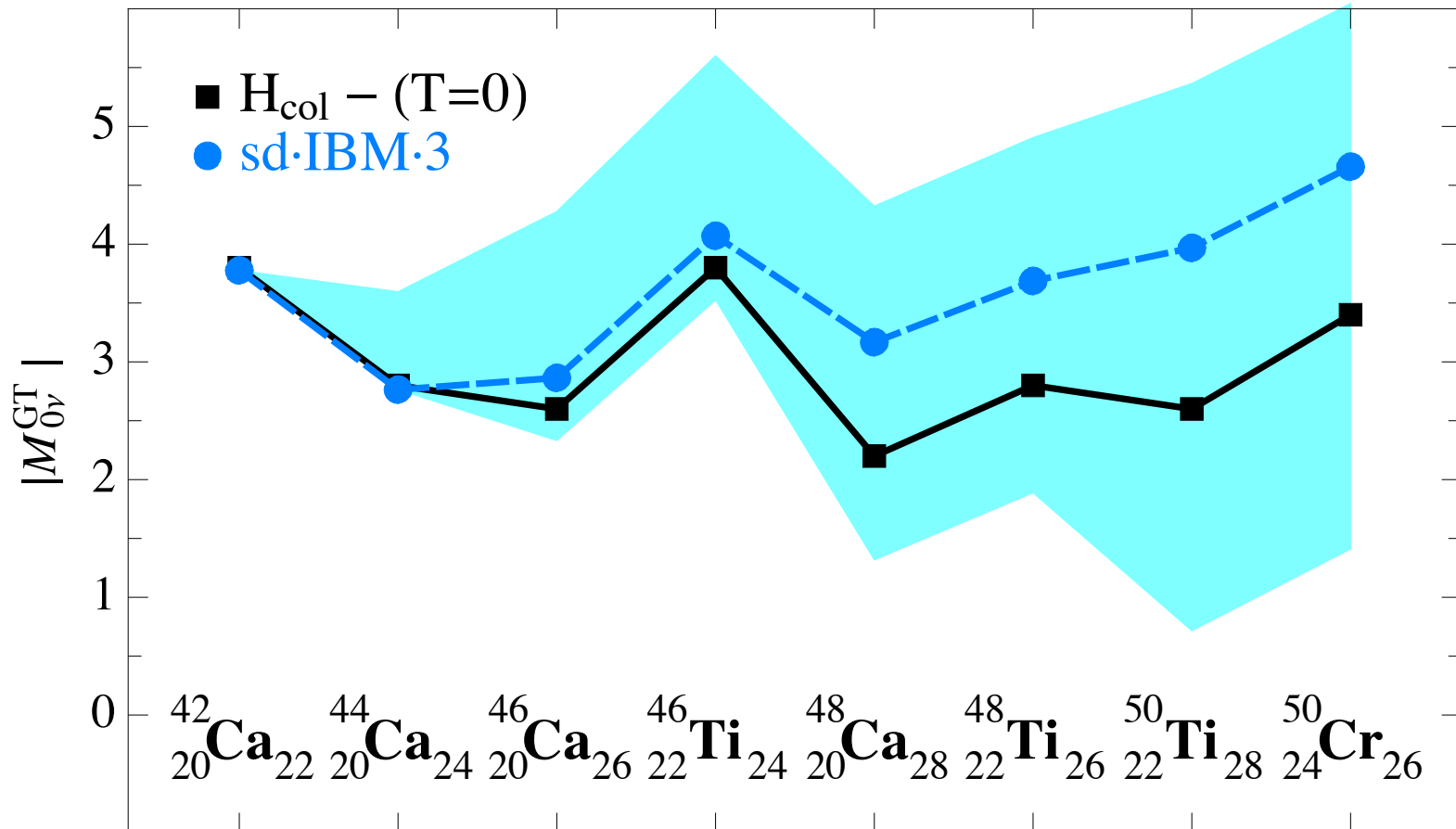
Gamow-Teller $0\nu\beta\beta$ in *sd*-IBM-3



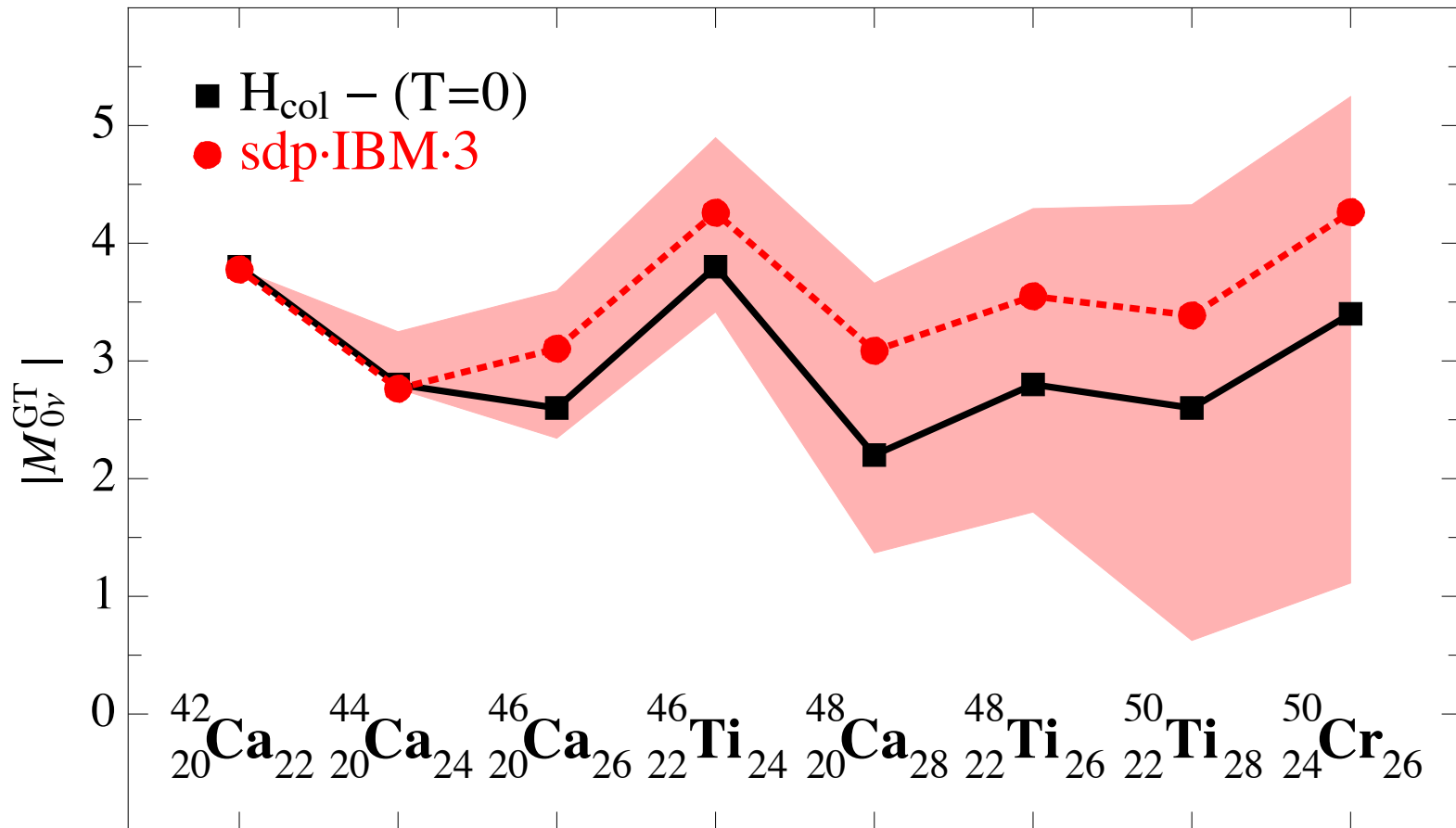
Gamow-Teller $0\nu\beta\beta$ in *sdp*-IBM-3



Gamow-Teller $0\nu\beta\beta$ in *sd*-IBM-3



Gamow-Teller $0\nu\beta\beta$ in *sdp*-IBM-3



Conclusions and outlook

Okubo-Lee-Suzuki transformation is used to define an effective collective Hamiltonian.

In light $\beta\beta$ emitters (^{48}Ca , ^{76}Ge , ^{82}Se):

Isospin invariance must be included in the IBM.

Isoscalar-pair bosons are not needed for energy spectra but they seem important for $\beta\beta$ decay.

Questions/problems:

Boson-number dependence of the Hamiltonian?

Do we need operators renormalized to the collective space?

How to couple this to phenomenology?



Dear Franco,

Thank you for all these years of guidance and collaboration. These pictures are a reminder of one of the many times I had the pleasure to spend with you, at the occasion of our trip to Bayeux to see its tapestry, showing in pictures the invasion of England by the Normans and the battle of Hastings.

Vera and I wish you a long and happy retirement. I am sure you will remain active in research and look forward to exchanging fruitful ideas with you for many years to come.

Piet.



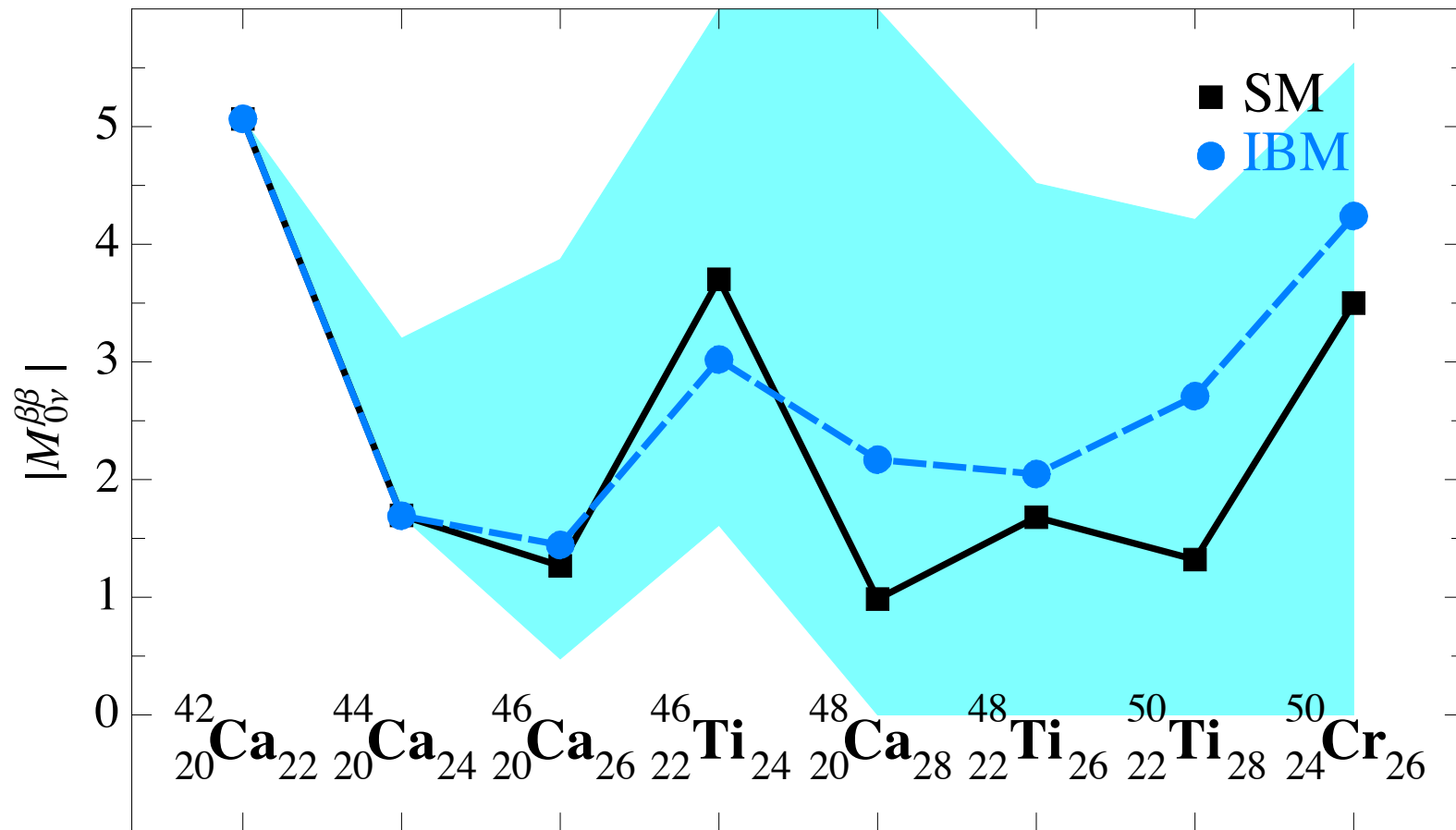
Observed $2\nu\beta\beta$ emitters

Isotope	isotopic abundance (%)	$Q_{\beta\beta}$ [MeV]
^{48}Ca	0.187	4.263
^{76}Ge	7.8	2.039
^{82}Se	9.2	2.998
^{96}Zr	2.8	3.348
^{100}Mo	9.6	3.035
^{116}Cd	7.6	2.813
^{130}Te	34.08	2.527
^{136}Xe	8.9	2.459
^{150}Nd	5.6	3.371

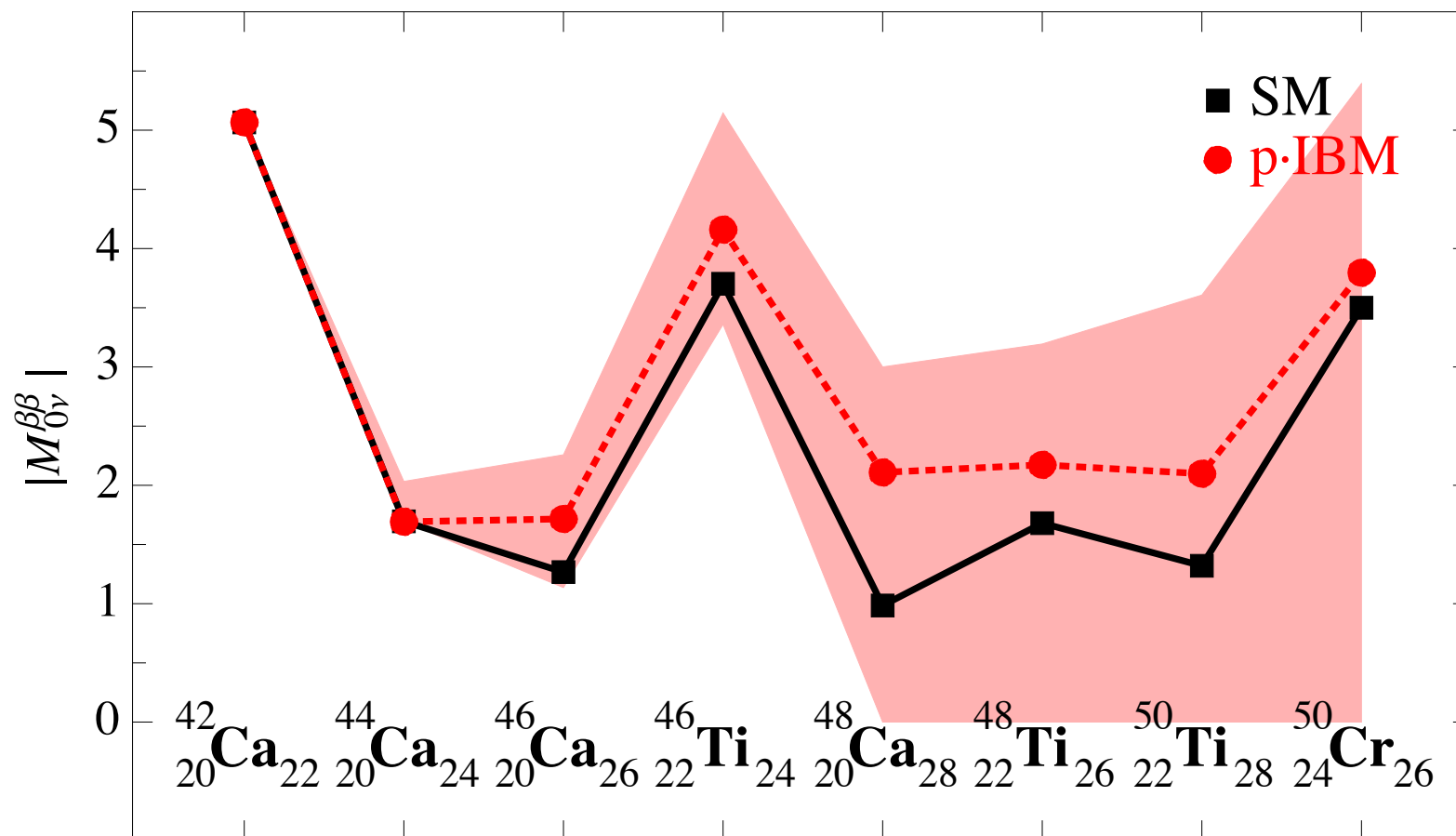
$0\nu\beta\beta$ experiments

Isotope	Technique	$T_{1/2}^{0\nu}$	$\langle m_{\beta\beta} \rangle$ (eV)	Reference
^{48}Ca	CaF_2 scint. crystals	$>1.4 \times 10^{22}$ yr	$<7.2-44.7$	Ogawa <i>et al.</i> (2004)
^{76}Ge	$^{\text{enr}}\text{Ge}$ det.	$>1.9 \times 10^{25}$ yr	<0.35	Klapdor-Kleingrothaus <i>et al.</i> (2001a)
^{76}Ge	$^{\text{enr}}\text{Ge}$ det.	$(2.23^{+0.44}_{-0.31}) \times 10^{25}$ yr (1σ)	0.32 ± 0.03	Klapdor-Kleingrothaus and Krivosheina (2006)
^{76}Ge	$^{\text{enr}}\text{Ge}$ det.	$>1.57 \times 10^{25}$ yr	$<0.33-1.35$	Aalseth <i>et al.</i> (2002a)
^{82}Se	Thin metal foils and tracking	$>2.1 \times 10^{23}$ yr	$<1.2-3.2$	Barabash (2006b)
^{100}Mo	Thin metal foils and tracking	$>5.8 \times 10^{23}$ yr	$<0.6-2.7$	Barabash (2006b)
^{116}Cd	$^{116}\text{CdWO}_4$ scint. crystals	$>1.7 \times 10^{23}$ yr	<1.7	Danevich <i>et al.</i> (2003)
^{128}Te	Geochemical	$>7.7 \times 10^{24}$ yr	$<1.1-1.5$	Bernatowicz <i>et al.</i> (1993)
^{130}Te	TeO_2 bolometers	$>3.0 \times 10^{24}$ yr	$<0.41-0.98$	Arnaboldi <i>et al.</i> (2007)
^{136}Xe	Liquid Xe scint.	$>4.5 \times 10^{23}$ yr ^a	$<0.8-5.6$	Bernabei <i>et al.</i> (2002)
^{150}Ne	Thin metal foils and tracking	$>3.6 \times 10^{21}$ yr		Barabash (2005)

Total $0\nu\beta\beta$ in *sd*-IBM



Total $0\nu\beta\beta$ in *sdp*-IBM



The mapped $0\nu\beta\beta$ operator

