

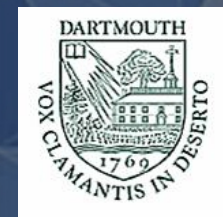
SYMMETRIES AND ORDER: ALGEBRAIC METHODS IN MANY BODY PHYSICS

October 6, 2018, Yale University

My journey through symmetries

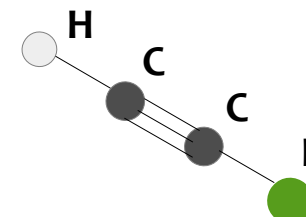
Lorenza Viola

Department of Physics & Astronomy
Dartmouth College



Back to the beginning...

“Spettroscopia roto-vibrazionale di molecole lineari quadriatomiche nel modello a vibroni: monofluoroacetilene” (Laurea dissertation, Trento, Dec 1991)



MOLECULAR PHYSICS, 1993, VOL. 78, No. 3, 545–559

Vibrational analysis of monofluoroacetylene (HCCF) in the vibron model

By F. IACHELLO

Center for Theoretical Physics, Sloane Laboratory, Yale University, New Haven, Connecticut 06511, USA

S. OSS

Dipartimento di Fisica, Università di Trento, 38050 Povo, Italy, and INFN, Gruppo Collegato di Trento, Italy

and L. VIOLA

Dipartimento di Fisica, Università di Trento, 38050 Povo, Italy

MOLECULAR PHYSICS, 1993, VOL. 78, No. 3, 561–575

Rotation–vibration interaction and Fermi resonances of HCCF in the vibron model

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Algebraic approach to molecular rotation-vibration spectra: Rotation-vibration interactions

J. Chem. Phys. 101 (5), 1 September 1994

Francesco Iachello

Center for Theoretical Physics, Yale University, New Haven, Connecticut 06511

Stefano Oss

Dipartimento di Fisica, Università di Trento, 38050 Povo, Italy

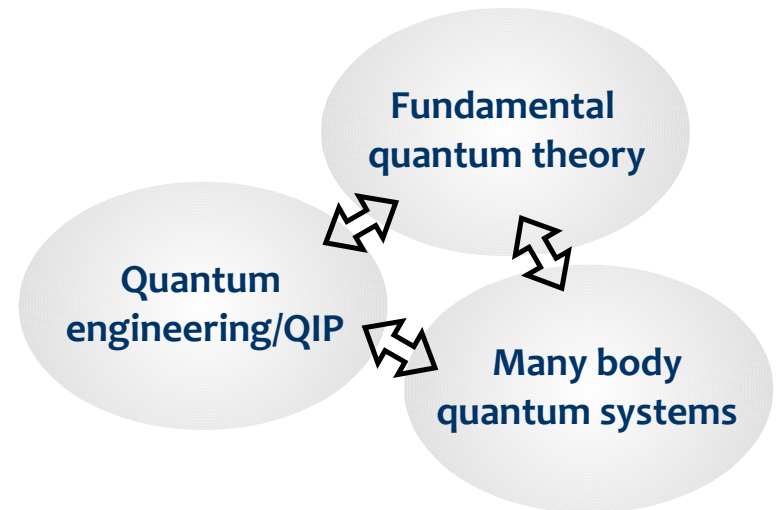
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Fast-forward to today...

Symmetries have remained a recurring theme in terms of both providing guiding physical principles and powerful mathematical tools...

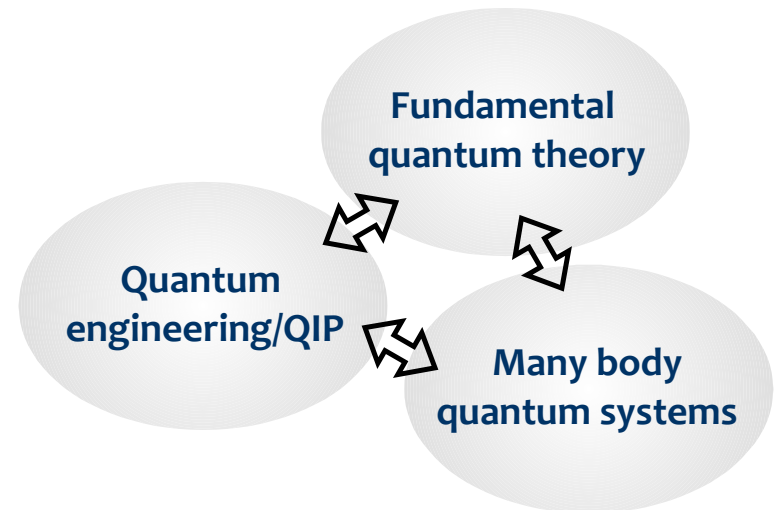
- Broad goal: To explore opportunities (and limitations) in modeling and controlling realistic, 'complex' quantum-dynamical systems
 - To what extent (how) can external control modify *seemingly irreversible* dynamical behavior?
 - How can control be leveraged to achieve *robust implementations* of quantum information processing?
 - How do we understand and harness quantum coherence and *entanglement*?
 - To what extent (how) can we *classify and engineer* quantum states of matter?...



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This talk: Snapshots of selected applications

- I. Open quantum systems – Noise suppression by symmetry design...
- II. Quantum phase transitions – Entanglement from symmetries, and beyond...

Passive error control via noiseless subsystems

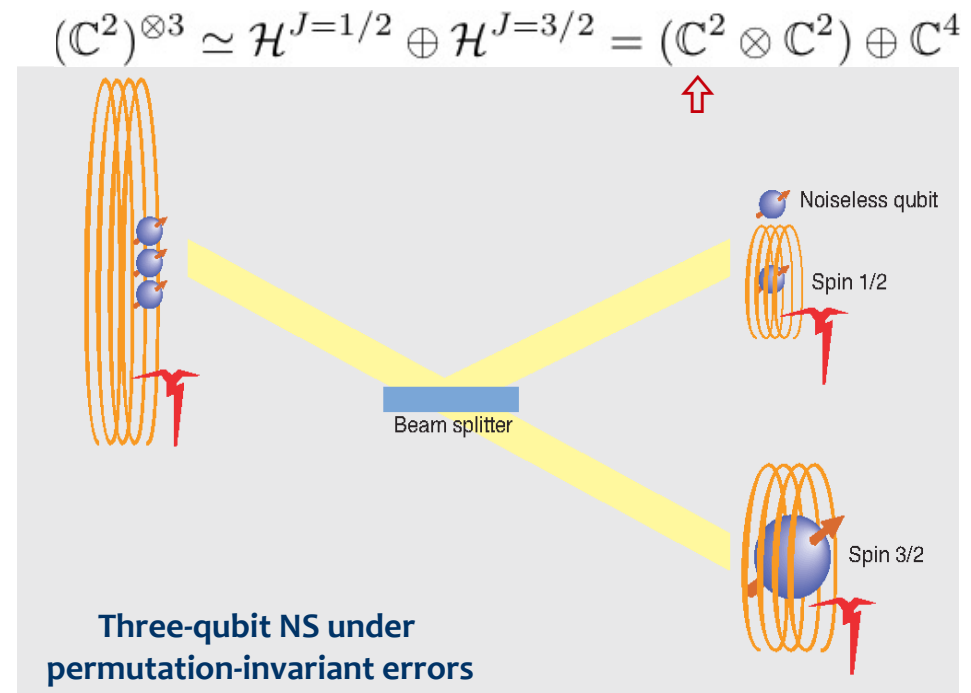
Knill, Laflamme & LV, *Theory of quantum error correction for general noise*, PRL 84 (2000); LV et al, *Experimental realization of noiseless subsystems for quantum information processing*, Science 293 (2001).

Key insight: Exploit symmetries that may naturally occur in the noise process.

- Look for *degenerate* error action: formal characterization based on representation theory of associative *-algebras:

$$\mathcal{H} \simeq \bigoplus_{\ell} \left(\mathcal{H}_{\ell}^{(L)} \otimes \mathcal{H}_{\ell}^{(Z)} \right) \oplus \mathcal{H}_R$$

$$E_a \simeq \bigoplus_{\ell} \left(\mathbb{I}_{\ell}^{(L)} \otimes \text{Mat}_{\mathbb{C}}^{(Z)}(d_{\ell}) \right) \oplus \mathbb{O}_R$$



Passive error control via noiseless subsystems

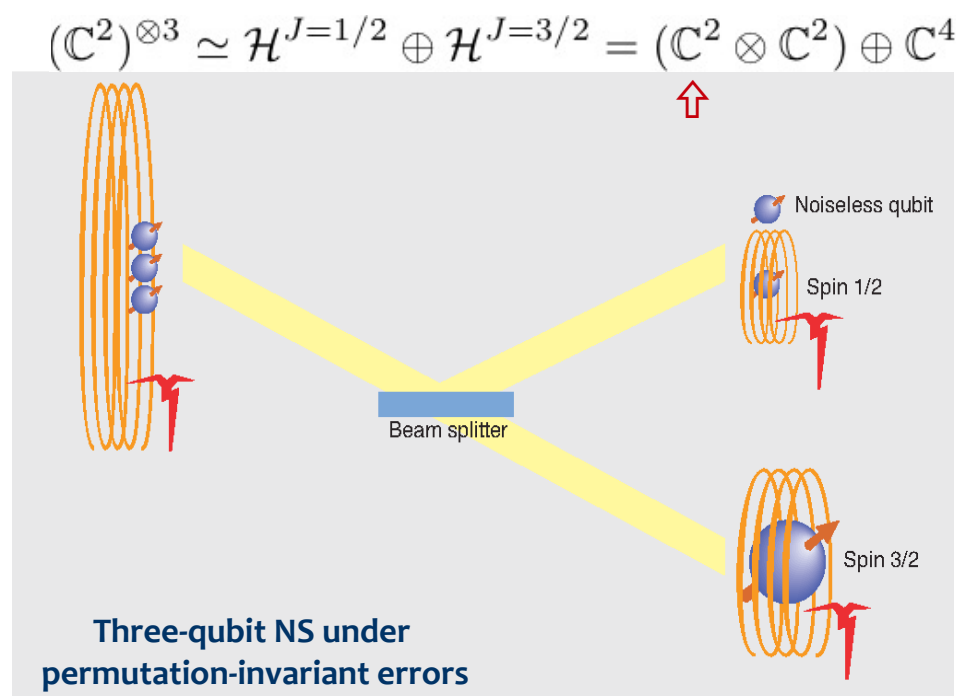
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- Once encoded in a noiseless subsystem, information is protected *indefinitely in time*
 - 'Infinite-distance' quantum error correction, no active recovery needed.
 - Subsystems provide *most general, unifying approach* to encoding quantum information.

Active error control via dynamical symmetrization

LV, Knill, & Lloyd, *Dynamical decoupling of open quantum systems*, PRL 82 (1999);
Khodjasteh & LV, *Dynamically error-corrected gates for universal quantum computation*, *ibid.* 102 (2009).

Key insight: Engineer controlled dynamics with desired symmetry properties.

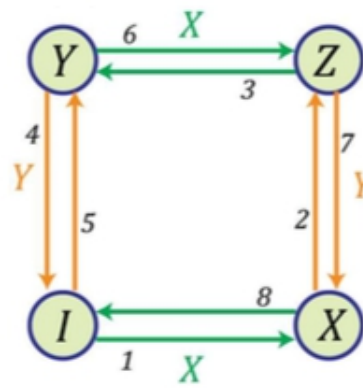
- If 'fast' external control is available, ensure that 'slow' unwanted dynamics is *filtered out* by the symmetry that control imposes:

$$H_{\text{tot}}(t) = (H_S + H_{\text{ctrl}}(t)) \otimes \mathbb{I}_B + \mathbb{I}_S \otimes H_B + \sum_a E_a \otimes B_a \Rightarrow U_{\text{tot}}(T) \approx U_{\text{target}} \otimes U_B$$

→ Control is designed starting from a *decoupling group* $\mathcal{G} \equiv \{g_\ell\}$, $\ell = 1, \dots, |\mathcal{G}|$

$$\begin{aligned}\Pi_{\mathcal{G}}(E_a) &\equiv \frac{1}{|\mathcal{G}|} \sum_{\ell} U_{g_\ell}^\dagger E_a U_{g_\ell} \\ &= 0, \quad \forall E_a\end{aligned}$$

- Significant error suppression can be achieved for realistic 'non-Markovian' noise environments.



$U_{\text{target}} = \mathbb{I}$ Dynamical decoupling

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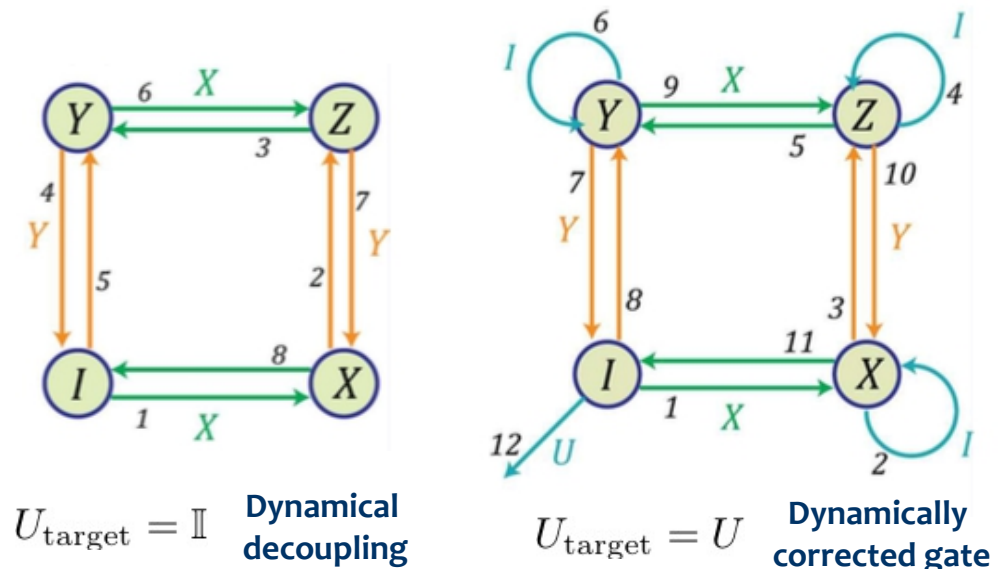
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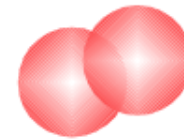


Generalized entanglement from Lie algebras

Barnum, Knill, Ortiz, Somma & LV, *A subsystem-independent generalization of entanglement*, PRL 92 (2004).



$$|\Psi\rangle = \frac{|\uparrow\rangle_A \otimes |\downarrow\rangle_B - |\downarrow\rangle_A \otimes |\uparrow\rangle_B}{\sqrt{2}}$$



$$\langle \vec{r}_1, \vec{r}_2 | \Psi_F \rangle = \frac{1}{\sqrt{2}} \begin{vmatrix} \phi_\alpha(\vec{r}_1) & \phi_\beta(\vec{r}_2) \\ \phi_\beta(\vec{r}_1) & \phi_\alpha(\vec{r}_2) \end{vmatrix}$$

- Standard notion of entanglement is inapplicable to systems of indistinguishable particles...

Key insight: Define entanglement relative to a preferred set of observables.

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- Standard notion of entanglement is inapplicable to systems of indistinguishable particles...

Key insight: Define entanglement relative to a preferred set of observables.

- If the preferred set is a semisimple, irreducibly represented Lie algebra $\mathfrak{h}$, a pure state is *generalized unentangled* relative to $\mathfrak{h}$ iff it is a *generalized coherent state* of $\mathcal{G} = e^{i\mathfrak{h}}$.

$$\mathfrak{h} \equiv \text{span}\{X_a\}, \text{Tr}(X_a X_b) = \delta_{ab} \Rightarrow \begin{aligned} P_{\mathfrak{h}}(|\psi\rangle) &\equiv \text{Tr}(\rho_{\mathfrak{h}}^2) \propto \sum_{a, \text{traceless}} |\langle \psi | X_a | \psi \rangle|^2 \\ \text{GE}(|\psi\rangle) &\equiv 1 - P_{\mathfrak{h}}(|\psi\rangle) \end{aligned}$$



Generalized entanglement in broken-symmetry QPTs

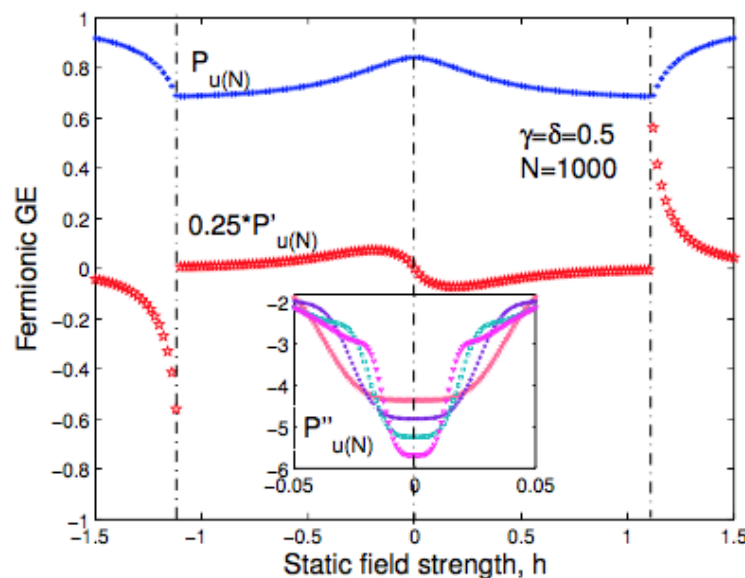
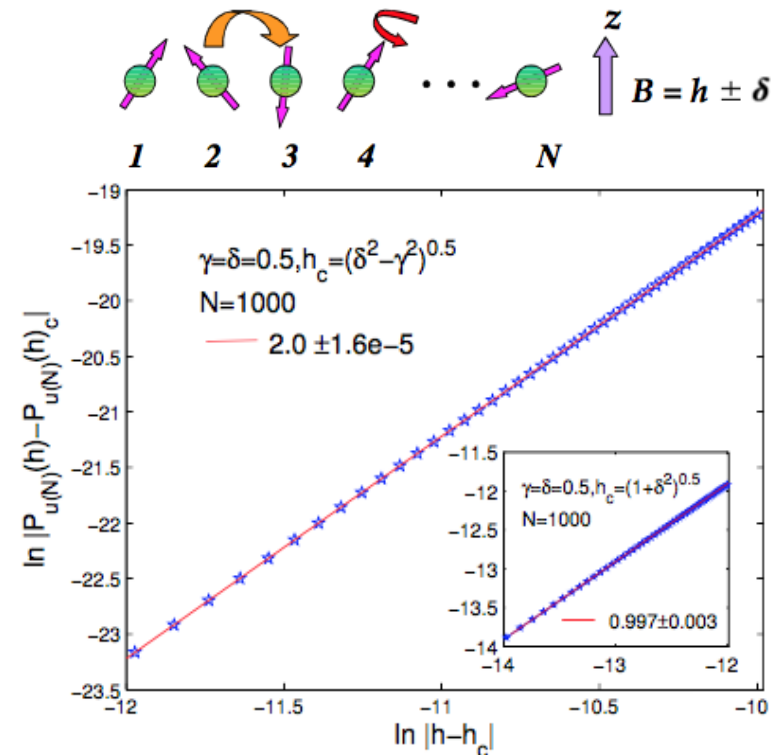
Somma, Ortiz, Barnum, Knill, & LV, *Nature and measure of entanglement in quantum phase transitions*, PRA 70 (2004); Deng et al, *Adv. Recent Progress Many-Body Theories* 9 (2006).

- Natural measure of *many-fermion entanglement* may be constructed from Lie algebra of single-particle (number-preserving) observables

$$\mathfrak{h}_F = \mathfrak{u}(N) = \text{span}\{c_i^\dagger c_j\}, \quad 1 \leq i < j \leq N$$

Fermionic GE provides new perspectives and diagnostic tools in probing QPTs.

Case study: Anisotropic XY chain in alternating field

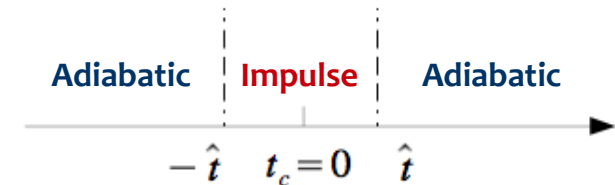


Generalized entanglement dynamics in broken-symmetry QPTs

Deng, LV & Ortiz, *Generalized entanglement in static and dynamic quantum phase transitions*,
in: Adv. Recent Progress Many-Body Theories 11 (2009).

- Simplest dynamical scenario: Linear parameter quench across a quantum critical point
 - *Kibble-Zurek mechanism*: Final density of excitation may be predicted on the basis of equilibrium critical exponents

$$\hat{t} \sim \tau_Q^{\nu z / (\nu z + 1)} \Rightarrow n(t_F) \sim \tau_Q^{-\nu / (\nu z + 1)}$$



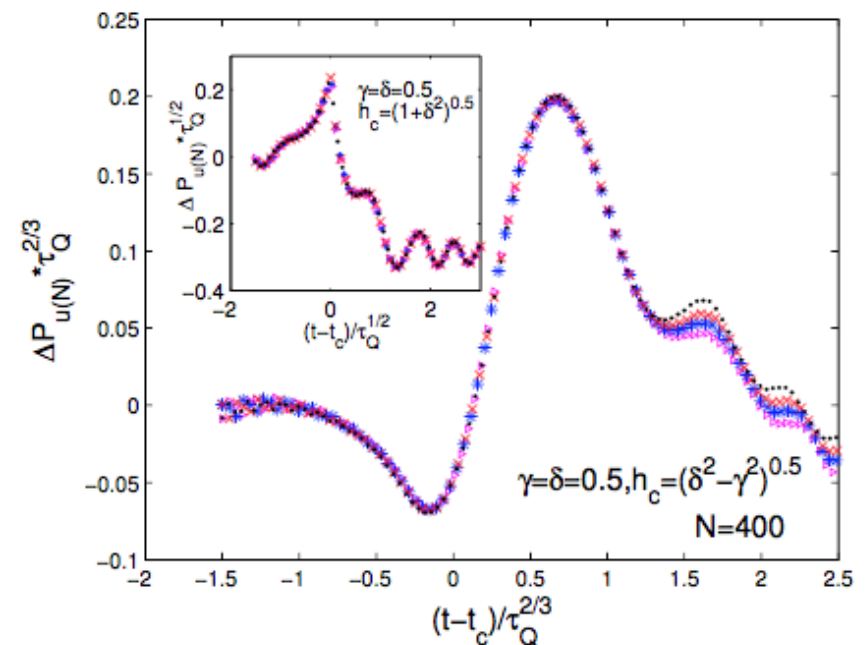
- Fermionic GE relative to the instantaneous ground state obeys *universal scaling behavior* throughout the entire quench dynamics

$$h(t) - h_c = (t - t_c) / \tau_Q$$

$$\begin{aligned} \Delta P_{u(N)}(t) &\equiv P_{u(N)}(|\Psi(t)\rangle) - P_{u(N)}(|\Psi_{GS}(t)\rangle) \\ &= \tau_Q^{-\nu / (\nu z + 1)} F\left(\frac{t - t_c}{\hat{t}}\right) \end{aligned}$$

- Scaling exponent agrees with KZM prediction
[Caveat: gapped phases, no multicriticality].

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Topological QPTs via chiral-symmetry engineering

Poudel, Ortiz & LV, *Dynamical generation of Floquet Majorana flat bands in s-wave superconductors*, EPL 110 (2015).

Use control to access 'synthetic' quantum phases that need not exist at equilibrium.

- *Chiral symmetries* play a key role in determining existence and stability of Majorana edge modes in topologically non-trivial superconducting phases:

$$[H, \mathcal{K}]_+ = 0, \quad \mathcal{K}^\dagger = \mathcal{K}^{-1}$$

- Control may be used (1) to 'activate' a chiral symmetry already existing at equilibrium; or (2) to 'generate' a chiral symmetry that is broken at equilibrium...

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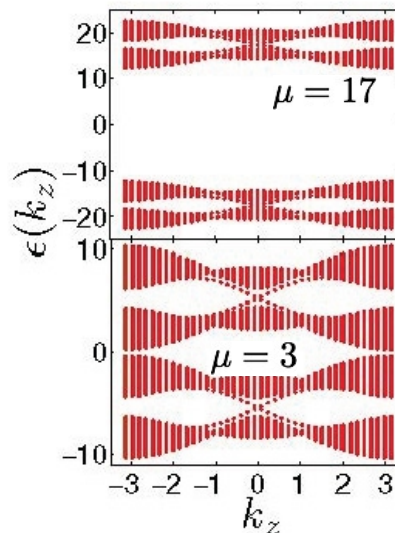
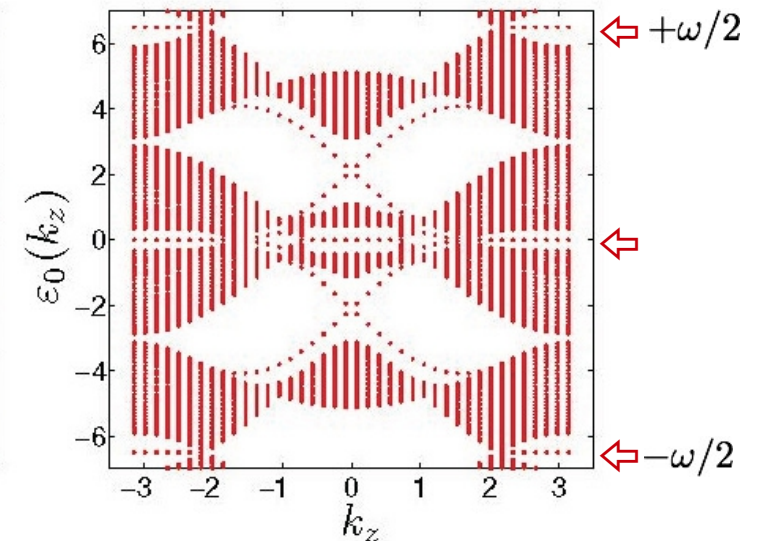
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- Control may be used (1) to 'activate' a chiral symmetry already existing at equilibrium; or (2) to 'generate' a chiral symmetry that is broken at equilibrium...

- Floquet Majoranas emerge at both zero *and* non-zero energy ($\Rightarrow$ no equilibrium counterpart) and are protected by activated symmetry.



Conclusion: Where to next?...

- Further roles and implications of symmetry in ongoing research:
 - Generalized Bloch theorem for mildly broken lattice translational symmetry...
Alase, Cobanera, Ortiz & LV, PRL 117 (2016); PRB 96 (2017); arXiv:1808.07555...
 - Generalized entanglement in relativistic settings, axiomatic quantum field theories...
 - Symmetry constraints in dissipative quantum control and quantum reservoir engineering...
- Lessons learned: (1) Symmetry in the quantum world may come in many flavors...
(2) '*Symmetry is a complexity-reducing concept... Seek it everywhere.*'
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Grazie – Thank you, Franco!